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HEAT CONDUCTION AND HEAT EXCHANGE IN TECHNOLOGICAL PROCESSES

MATHEMATICAL MODELING OF THE PROCESS OF WARMING UP OF A CYLINDRICAL SURFACE BY A MOVING INTENSE HEAT SOURCE

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A mathematical model of the process of warming up of a metal cylindrical surface by a moving point heat source has been proposed; the model allows for the presence of a thin absorbing coating on the metal surface and for the thermal resistance of the contact surface. An algorithm of calculation of a nonstationary temperature field has been constructed.; this algorithm makes it possible to find the values of the problem's parameters ensuring local surface heating. An example of numerical calculation has been given.

One widespread form of laser processing of metals is laser surface hardening involving the action of an intense laser-radiation flux on a local portion of the surface. A rapid heating of this portion to high temperatures results, after which the heated portion of the surface is cooled by heat conduction deep into the material and by heat transfer from the surface, once the action of the radiation has ceased. Information on the thermal state of the metal in the laser processing is a source for analysis of the dimensions of the heat-affected zone and the properties of the surface hardened [1].

We consider the process of warming up of a hollow cylinder by a point source following a spiral path on the exterior cylinder surface with a high velocity. Considering the angle of the spiral line as being fairly small, we may assume that the heat source moves in a circle lying in a plane which is perpendicular to the generatrix of the cylinder. In such a model, the process of distribution of heat in the hollow cylinder may be considered for the case of its heating from a lumped ring source with a heat-flux density of $q_*\delta(z - z_*(t))$, where q_* = const, moving on the exterior lateral surface. In the process of heating of the cylinder by such a heat source, we have heat transfer from the interior and exterior cylindrical surfaces to the environment by the Newton law and heat exchange by radiation on the exterior lateral surface.

Thus, determination of the temperature field in the hollow cylinder with a moving local ring heat source contributes to the solution of the initial boundary-value problem for the heat-conduction equation. This problem assumes a more general character if the presence of a thin absorbing coating on the metal surface [2] and the thermal resistance of the contact surface between the metal and the absorbing coating are allowed for. Furthermore, allowance for the dependence of thermophysical parameters on temperature requires that the formulation of the problem be nonlinear. As a result we arrive at the following mathematical model of the process in question:

$$\rho_j c_j (T_j) \frac{\partial T_j}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(\lambda_j (T_j)\, r \frac{\partial T_j}{\partial r} \right) + \frac{\partial}{\partial z} \left(\lambda_j (T_j) \frac{\partial T_j}{\partial z} \right), \quad t > 0 , \quad r_{j-1} < r < r_j , \quad 0 < z < h , \quad j = 1, 2 ; \tag{1}$$

on the exterior coating surface $r = r_2$, the condition of heat exchange will take the form

$$\lambda_2 (T_2) \frac{\partial T_2}{\partial r} \bigg|_{r=r_2} = q_* \delta (z - z_* (t)) + \alpha_2 (T_{\text{env}} - T_2 (r_2, z, t)) + \sigma\varepsilon (T_{\text{env}}^4 - T_2^4 (r_2, z, t)) , \quad t > 0 , \quad 0 \le z \le h ; \tag{2}$$

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on the interior surface of the hollow metal cylinder $r = r_0$, the condition of heat exchange will be written as

$$\lambda_1 (T_1) \left.\frac{\partial T_1}{\partial r}\right|_{r=r_0} = \alpha_1 (T_1 (r_0, z, t) - T_{\text{env}}) , \quad t > 0 , \quad 0 \le z \le h ; \tag{3}$$

the end surfaces $z = 0$ and $z = h$ of the hollow metal cylinder and the cylindrical shell of the coating are set heat-insulated:

$$\left.\frac{\partial T_j}{\partial z}\right|_{\substack{z=0,\\ z=h}} = 0 , \quad t > 0 , \quad r_{j-1} \le r \le r_j , \quad j = 1, 2 ; \tag{4}$$

the condition on the contact surface $r = r_1$ is the condition of nonideal thermal contact

$$\lambda_1 (T_1) \left.\frac{\partial T_1}{\partial r}\right|_{r=r_1} = \frac{1}{R}\Big(T_2 (r_1, z, t) - T_1 (r_1, z, t)\Big) = \lambda_2 (T_2) \left.\frac{\partial T_2}{\partial r}\right|_{r=r_1} , \quad t \ge 0 , \quad 0 \le z \le h ; \tag{5}$$

the metal and coating temperature at the initial instant of time will be taken to be constant and equal to the environmental temperature:

$$T_j (r, z, 0) = T_{\text{env}} , \quad r_{j-1} \le r \le r_j , \quad 0 \le z \le h . \tag{6}$$

The mathematical model (1)–(6) is easily generalized to the case where the ring heat source executes periodic reciprocating motion (scanning) with a constant velocity V_* along the cylinder axis, and the law of motion $z_*(t)$ may be prescribed in the following manner:

$$z_* (t) = \begin{cases} l + V_* (t - t_{i-1}) , & i = 1, 3, 5, ... ; \\ h - l - V_* (t - t_{i-1}) , & i = 2, 4, 6, ... ; \end{cases} \quad t_{i-1} \le t \le t_i , \tag{7}$$

where $t_i = it_*$, $t_* = (h - 2l)/V_*$, and $l < h/2$.

The present work seeks to determine the problem's parameters q_* and V_* for which a limiting temperature T_{lim} no higher than the melting temperature of the metal is attained in the surface layer of the hollow metal cylinder. Below, we propose a modification of the method [3, 4] for finding the approximate analytical solution of problem (1)–(6).

We introduce the functions

$$C_j (T_j, r) = \rho_j r c_j (T_j) , \quad \Lambda_j (T_j, r) = r\lambda_j (T_j) , \quad j = 1, 2 ;$$

into consideration; then Eqs. (1) will take the form

$$C_j (T_j, r) \frac{\partial T_j}{\partial t} = \text{div}\Big(\Lambda_j (T_j, r) \text{ grad } T_j\Big), \quad t > 0 , \quad r_{j-1} < r < r_j , \quad 0 < z < h , \quad j = 1, 2 , \tag{8}$$

the variables r and z here should be interpreted as orthogonal coordinates.

We discretize the time variable t by the system of points $t_k = k\tau$, $k = 1, 2, ...$, with a fairly small step $\tau > 0$ and replace the time derivatives in Eqs. (8) by the finite-difference relations

$$\left.\frac{\partial T_j}{\partial t}\right|_{t=t_k} \approx \frac{T_j^{(k)} (r, z) - T_j^{(k-1)} (r, z)}{\tau} , \quad k = 1, 2, 3, ... , \quad j = 1, 2 ,$$

where $T_j^{(k)}(r, z)$ are the approximate values of the functions $T_j(r, z, t)$ at $t = t_k$.

Next we linearize Eqs. (8), setting the coefficients C_j and Λ_j on each time layer $t = t_k$ the known functions computed on the previous time layer $t = t_{k-1}$. We introduce the notation

$$C_j^{(k)}(r, z) = C_j\left(T_j^{(k-1)}(r, z), r\right), \quad \Lambda_j^{(k)}(r, z) = \Lambda_j\left(T_j^{(k-1)}(r, z), r\right), \quad j = 1, 2 .$$

Furthermore, on each time layer $t = t_k$, we will also assume the heat fluxes in conjugation condition (5) to be known and equal to $\frac{1}{R}(T_2^{(k-1)}(r_1, z) - T_1^{(k-1)}(r_1, z))$. This makes it possible to write the differential-difference analog of problem (1)–(6) in the form of the following iteration scheme ($k = 1, 2, ...$) of solution of two boundary-value problems for linear elliptic equations with variable coefficients $\Lambda_j^{(k)}(r, z)$ and $C_j^{(k)}(r, z)$ for the functions $T_j^{(k)}(r, z)$ sought:

$$\text{(I)} \quad -\operatorname{div}\left(\Lambda_1^{(k)}(r, z)\operatorname{grad} T_1^{(k)}(r, z)\right) + \frac{1}{\tau} C_1^{(k)}(r, z)\, T_1^{(k)}(r, z) =$$

$$= \frac{1}{\tau} C_1^{(k)}(r, z)\, T_1^{(k-1)}(r, z), \quad r_0 < r < r_1, \quad 0 < z < h ; \tag{9}$$

$$\Lambda_1^{(k)}(r, z)\left.\frac{\partial T_1^{(k)}}{\partial r}\right|_{r=r_0} = Q_0^{(k)}(z), \quad 0 \le z \le h ; \quad \Lambda_1^{(k)}(r, z)\left.\frac{\partial T_1^{(k)}}{\partial r}\right|_{r=r_1} = Q_1^{(k)}(z), \quad 0 \le z \le h ;$$

$$\left.\frac{\partial T_1^{(k)}}{\partial z}\right|_{z=0,\, z=h} = 0, \quad r_0 \le r \le r_1 ; \tag{10}$$

$$\text{(II)} \quad -\operatorname{div}\left(\Lambda_2^{(k)}(r, z)\operatorname{grad} T_2^{(k)}(r, z)\right) + \frac{1}{\tau} C_2^{(k)}(r, z)\, T_2^{(k)}(r, z) =$$

$$= \frac{1}{\tau} C_2^{(k)}(r, z)\, T_2^{(k-1)}(r, z), \quad r_1 < r < r_2, \quad 0 < z < h ; \tag{11}$$

$$\Lambda_2^{(k)}(r, z)\left.\frac{\partial T_2^{(k)}}{\partial r}\right|_{r=r_1} = Q_1^{(k)}(z), \quad 0 \le z \le h ; \quad \Lambda_2^{(k)}(r, z)\left.\frac{\partial T_2^{(k)}}{\partial r}\right|_{r=r_2} = Q_2^{(k)}(z), \quad 0 \le z \le h ;$$

$$\left.\frac{\partial T_2^{(k)}}{\partial z}\right|_{z=0,\, z=h} = 0, \quad r_1 \le r \le r_2 . \tag{12}$$

Here we have

$$Q_0^{(k)}(z) = \alpha_1 r_0\left(T_1^{(k-1)}(r_0, z) - T_{\text{env}}\right), \quad Q_1^{(k)}(z) = \frac{r_1}{R}\left(T_2^{(k-1)}(r_1, z) - T_1^{(k-1)}(r_1, z)\right), \quad z_*^{(k)} = z_*(t_k),$$

$$Q_2^{(k)}(z) = q_* r_2 \delta\left(z - z_*^{(k)}\right) + r_2\left(\alpha_2 + 4\sigma\varepsilon T_{\text{env}}^3\right)\left(T_{\text{env}} - T_2^{(k-1)}(r_2, z)\right).$$

We note that in $Q_2^{(k)}(z)$, we have allowed for the approximation of the radiation condition

$$\sigma\varepsilon\left(T_{\mathrm{env}}^4-\left(T_2^{(k-1)}(r_2, z)\right)^4\right)\approx 4\sigma\varepsilon T_{\mathrm{env}}^3\left(T_{\mathrm{env}}-T_2^{(k-1)}(r_2, z)\right).$$

At the first iteration step, the functions $T_j^{(0)}(r, z)$ should be considered as being equal to the initial temperature T_{env} from conditions (6). At the kth iteration step, the functions $T_j^{(k)}(r, z)$ will be sought in the form of expansion in double trigonometric Fourier series

$$T_j^{(k)}(r, z)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\gamma_{mn}a_{j,mn}^{(k)}X_{j,mn}(r, z)\,,\quad j=1, 2\,;\quad \gamma_{mn}=\gamma_m\gamma_n\,,\quad \gamma_m=\begin{cases}0.5\,, & m=0\,,\\ 1\,, & m>0\,,\end{cases} \tag{13}$$

in total and orthogonal systems of eigenfunctions

$$X_{j,mn}(r, z)=\cos\left(\mu_{j,m}(r-r_{j-1})\right)\cos(\nu_n z)\,,\quad \mu_{j,m}=\frac{m\pi}{r_j-r_{j-1}}\,,\quad \nu_n=\frac{n\pi}{h}\,,$$

of the following Sturm–Liouville problems:

$$\Delta X_j+\Theta_j X_j=0\,,\quad \left.\frac{\partial X_j}{\partial n}\right|_{\partial\Omega_j}=0\,,$$

where $\Delta=\dfrac{\partial^2}{\partial r^2}+\dfrac{\partial^2}{\partial z^2}$ is the Laplace operator and $\partial/\partial n$ is the normal derivative of the boundary of the domain

$$\Omega_j=\left\{(r, z):\ \ r_{j-1}<r<r_j\,,\ \ 0<z<h\right\}.$$

To find the Fourier coefficients $a_{j,mn}^{(k)}$, $j = 1, 2$, in expansions (13) we multiply Eq. (9) and (11) by the functions $X_{1,ps}(r, z)$ and $X_{2,ps}(r, z)$ respectively and thereafter integrate the resulting equalities over the domains Ω_1 and Ω_2. Carrying out transformations with account for boundary conditions (10) and (12), we arrive at an infinite system of linear algebraic equations for the sought Fourier coefficients $a_{j,mn}^{(k)}$

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}A_{j,psmn}^{(k)}\gamma_{mn}a_{j,mn}^{(k)}=b_{j,ps}^{(k)}\,,\quad p=0, 1, 2, \ldots\,,\quad s=0, 1, 2, \ldots\,,\quad j=1, 2\,, \tag{14}$$

$$A_{j,psmn}^{(k)}=\left(\mu_{j,m}\mu_{j,p}+\nu_n\nu_s\right)\tau\left(\xi_{j,(|m-p|,|n-s|)}^{(k)}-\xi_{j,(m+p,n+s)}^{(k)}\right)+\left(\mu_{j,m}\mu_{j,p}-\nu_n\nu_s\right)\tau\left(\xi_{j,(|m-p|,n+s)}^{(k)}-\xi_{j,(m+p,|n-s|)}^{(k)}\right)+$$
$$+\eta_{j,(|m-p|,|n-s|)}^{(k)}+\eta_{j,(m+p,|n-s|)}^{(k)}+\eta_{j,(|m-p|,n+s)}^{(k)}+\eta_{j,(m+p,n+s)}^{(k)}\,,$$

$$b_{j,ps}^{(k)}=f_j^{(k)}+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\gamma_{mn}a_{j,mn}^{(k-1)}\left(\eta_{j,(|m-p|,|n-s|)}^{(k)}+\eta_{j,(m+p,|n-s|)}^{(k)}+\eta_{j,(|m-p|,n+s)}^{(k)}+\eta_{j,(m+p,n+s)}^{(k)}\right),$$

$$f_1^{(k)}=\frac{8\tau}{r_1-r_0}\left[(-1)^p\,\theta_s^{(k)}-\varphi_s^{(k)}\right],\ \ f_2^{(k)}=\frac{8\tau}{r_2-r_1}\left[(-1)^p\,\psi_s^{(k)}-\theta_s^{(k)}\right],$$

where $\varphi_s^{(k)}$, $\theta_s^{(k)}$, and $\psi_s^{(k)}$ are the Fourier coefficients of the expansions of the functions $Q_0^{(k)}(z)$, $Q_1^{(k)}(z)$, and $Q_2^{(k)}(z)$ respectively in trigonometric series in the total and orthogonal (on the segment $0\le z\le h$) system of functions

TABLE 1. Thermophysical Parameters of the Materials of the Hollow Cylinder and the Absorbing Coating

Parameters	*T*, K									
	300	400	600	800	1000	1200	1400	1600	1800	2000
λ_1, W/(m·K)	48	47	41	37	32	23	21	20	—	—
c_1, J/(kg·K)	469	505	521	660	616	577	560	545	—	—
λ_2, W/(m·K)	32.0	28.0	20.0	16.0	7.5	6.4	5.6	5.7	5.8	6.9
c_2, J/(kg·K)	860	875	943	1020	1086	1102	1140	1160	1170	1178

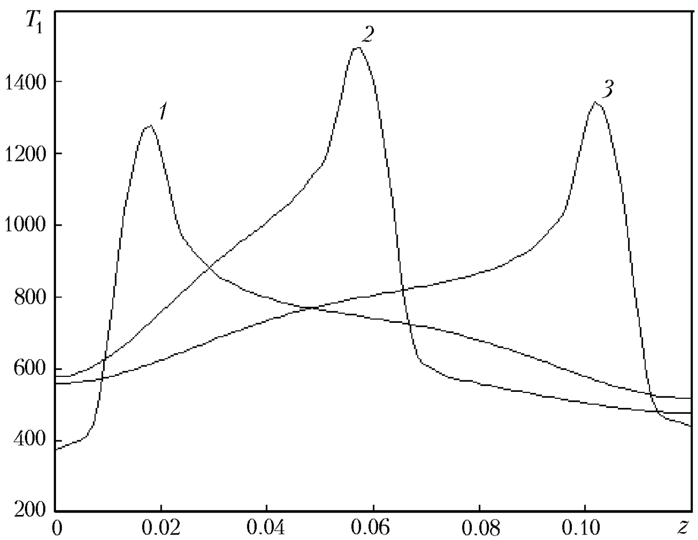

Fig. 1. Temperature distribution on the exterior surface $r = r_1$ of the hollow metal cylinder at the instants of time: 1) $t = 40$, 2) 50, 3) 60 sec. T_1, K; z, m.

$\left\{\cos(\nu_s z)\right\}_{s=0}^{\infty}$, $\xi_{j(p,s)}^{(k)}$ and $\eta_{j(p,s)}^{(k)}$ are the Fourier coefficients of the expansions of the functions $\Lambda_j^{(k)}(r, z)$ and $C_j^{(k)}(r, z)$ in double trigonometric series in the system $\left\{X_{j,ps}(r, z)\right\}_{p,s=0}^{\infty}$ in the domain Ω_j, $j = 1$ and 2.

We transform systems (14) to their standard form. For this purpose we number the elements of two-dimensional arrays $a_{j,,mn}^{(k)}$ and $b_{j,ps}^{(k)}$ by diagonals with equal sum of indices, establishing the correspondences $(p, s) \leftrightarrow v$ and $(m, n) \leftrightarrow w$ by the rules

$$v = \frac{1}{2}(p + s + 1)(p + s + 2) - s , \quad w = \frac{1}{2}(m + n + 1)(m + n + 2) - n ,$$

according to which we compose one-dimensional sequences β_w, $d_{j,w}^{(k)}$, and $g_{j,v}^{(k)}$ from the elements of two-dimensional arrays γ_{mn}, $a_{j,mn}^{(k)}$, and $b_{j,ps}^{(k)}$. Using the same rules, we compose two-dimensional arrays $D_{j,vw}^{(k)}$, $j = 1$ and 2, from the elements of multidimensional arrays $A_{j,psmn}^{(k)}$. As a result systems (14) will take the form

$$\sum_{w=1}^{\infty} D_{j,vw}^{(k)} \beta_w d_{j,w}^{(k)} = g_{j,v}^{(k)} , \quad v = 1, 2, \ldots , \quad j = 1, 2 . \tag{15}$$

It is noteworthy that, since the differential operator on the right-hand side of Eq. (8) is self-adjoint and positive definite, the matrices $\mathbf{D}_j^{(k)}$, $j = 1$ and 2, of systems (15) are symmetric and positive definite. In this case the solution of these infinite systems may be found by the reduction method [5], i.e., by solution of systems of finite order which are obtained from these infinite systems by truncation. In calculating, we determined the order of truncation of systems (15) on the basis of the Runge estimate [6].

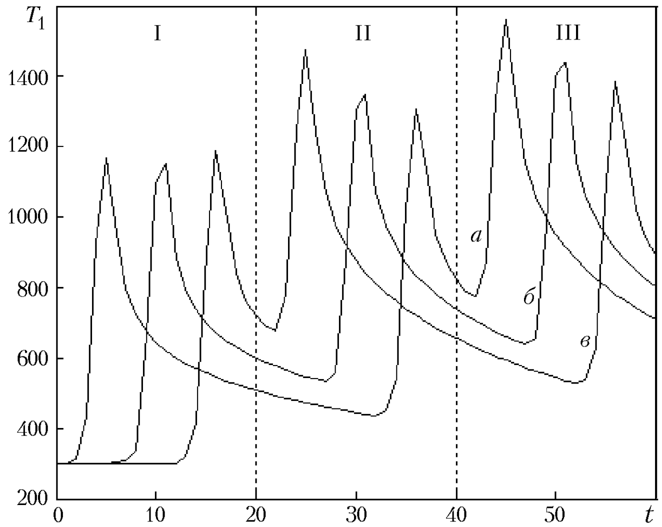

Fig. 2. Temperature on the metal surface $r = r_1$ vs. time for different values of z: 1) $z = z_1$; b) $z = z_2$; c) $z = z_3$. I–III, scanning steps. T_1, K; t, sec.

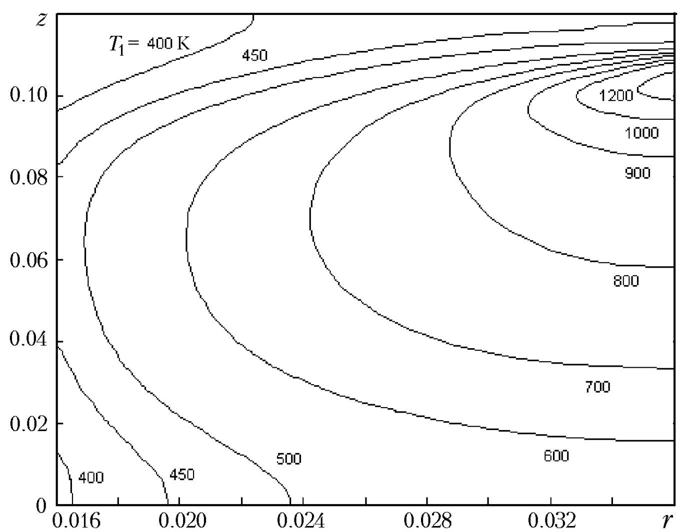

Fig. 3. Field of isotherms in the axial cross section of the cylinder at the third scanning step ($z_* = 0.105$ m). T_1, K; r, z, m.

As has been noted above, the present work seeks to determine such values of the problem's parameters q_* and V_*, for which we have

$$\max_{0 \le z \le h} T_1(r_1, z, t) \le T_{\lim}, \quad \forall t \cup [0, \tilde{t}],$$

where $\tilde{t}$ is a fixed value of time.

It is clear that the solution of this problem is not unique, i.e., there is a multitude of pairs of q_* and V_* values satisfying this condition. We give one solution, carrying out calculations for the following values of the parameters: $\rho_1 = 7780$ kg/m^3, $\rho_2 = 3000$ kg/m^3, $T_{\text{env}} = 300$ K, $T_{\lim} = 1600$ K, $\alpha_1 = 10^4$ W/(m^2·K), $\alpha_2 = 50$ W/(m^2·K), $R = 10^{-4}$ m^2·K/W, $\sigma = 5.67 \cdot 10^{-8}$ W/(m^2·K^4), $\varepsilon = 0.8$, $r_0 = 16 \cdot 10^{-3}$ m, $r_1 = 36 \cdot 10^{-3}$ m, $r_2 = 36.05 \cdot 10^{-3}$ m, $h = 120 \cdot 10^{-3}$ m, $l = 15 \cdot 10^{-3}$ m, and $\tilde{t} = 3t_*$.

The values of the thermal conductivities and specific heats of the materials of the hollow cylinder and the absorbing coating as functions of the temperature are indicated in Table 1 [7].

Varying the parameters q_* and V_* of the problem within the ranges $q_* = 10^6$–10^7 W/(m·K) and $V_* = (4$–$5)10^{-3}$ m/sec, we select values of $q_* = 8 \cdot 10^6$ W/(m·K) and $V_* = 4.5 \cdot 10^{-3}$ m/sec for which the maximum temperature in the surface layer of the hollow metal cylinder is no higher than the value of the limiting temperature T_{lim}:

$$\max_{0 \le z \le h} T_1 (r_1, z, t) \le T_{\text{lim}} = 1600 \text{ K} , \quad \forall t \cup [0, \tilde{t}] , \quad \tilde{t} = 60 \text{ sec} .$$

Figure 1 gives results of calculations of the temperature on the exterior surface $r = r_1$ of the hollow metal cylinder at fixed instants of time at the third scanning step. Figure 2 shows the evolution of the temperature on the metal surface $r = r_1$ for different values of z: $z_1 = 35 \cdot 10^{-3}$ m, $z_2 = 60 \cdot 10^{-3}$ m, and $z_3 = 85 \cdot 10^{-3}$ m. Figure 3 gives the field of isotherms in the axial cross section of the hollow cylinder at the third scanning step, when the heat source is at the point $z_* = 0.105$ m. As the results of calculations of the isotherm fields at different instants of time have shown, the depth of local heating of the hollow metal cylinder to a temperature of 1200–1600 K is no larger than $2 \cdot 10^{-3}$ m.

The rapid heating of a surface portion to high temperatures close to the value of the limiting temperature T_{lim} followed by its rapid cooling as a result of the heat transfer from the surface and heat conduction deep into the material makes it difficult to directly investigate such a process experimentally. Therefore, a numerical experiment is a necessity for investigating the thermal state of the metal and finding the problem's parameters that ensure local high-temperature surface heating. The proposed mathematical model of the nonstationary process of heat conduction in a two-layer hollow cylinder includes the nonlinear boundary condition and allows for the temperature dependence of the thermophysical parameters of the layer and for the presence of the thermal resistance of the contact surface between the metal and the absorbing coating. The constructed algorithm of calculation of the temperature field makes it possible to solve the problem under different heat-exchange conditions and for an arbitrary law of motion of the heat source.

NOTATION

c, specific heat, J/(kg·K); h, cylinder height, m; l, coordinate z of the ring heat source at the initial instant of time, m; R, thermal resistance, m^2·K/W; r, spatial coordinate along the cylinder radius, m; T, temperature, K; t, time, sec; t_*, time of one scanning step, sec; z, coordinate along the cylinder axis, m; $z_*(t)$, coordinates of the ring source along the cylinder axis; α, coefficient of convective heat transfer, W/(m^2·K); $\delta(z)$, delta function; ε, emissivity factor of the radiating surface; Θ, parameter of the Sturm–Liouville problem; λ, thermal conductivity, W/(m·K); ρ, density, kg/m^3; σ, Stefan–Boltzmann constant, W/($m^2 \cdot K^4$); τ, time step, sec. Subscripts and superscripts: 1, hollow cylinder; 2, absorbing coating; *, heat source; env, environment; i, scanning-step No.; k, iteration No.; lim, limiting.

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